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Hydromagnetic Stability of the Magnetospheric Boundary

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Abstract. The problem of the stability of the interface between the solar wind and the magnetosphere is investigated by means of hydromagnetic equations. An explicit criterion of instability is written for the special case in which both the solar plasma and the magnetospheric medium are supposed to be of the same density and carry a uniform magnetic field in the direction of streaming. It is concluded that the magnetospheric boundary is likely to be unstable toward the tail and under comparatively quiet solar conditions. It is suggested that the characteristic effects of instability should be observed toward the night side, preferably with space exploration vehicles.

AUTHOR

INTRODUCTION

It is now generally established that a continual expansion of the solar corona leads to a solar wind which blows radially into space [*Parker*, 1963]. The solar plasma interacts with the geomagnetic field, confining it in a region called the magnetosphere. The magnetosphere is asymmetrical in form and has the shape of a tear drop, with a 'tail' pointing away from the sun. An important question which has evoked some controversy in the literature is that of stability of the interface between the solar wind and the geomagnetic field. *Dungey* [1955] and *Parker* [1958] concluded on theoretical grounds that the interface is unstable. *Dessler* [1961], on the other hand, contends, on the basis of transient geomagnetic activity data obtained at the earth's surface from magnetometers, that the interface is stable. If the latter inference be correct, theories of aurora, magnetic storms, and Van Allen radiation which make use of the concept of turbulent solar injection need to be re-examined.

The purpose of the present paper is to investigate the hydromagnetic stability of a model of the interface between the solar wind and the magnetosphere. The solar wind, on impinging upon the magnetospheric boundary, slips past it, leading to a situation in which Kelvin-Helmholtz instability should play an important role. We regard both the solar beam and the magnetospheric medium as uniform infinitely extended plasmas of zero dissipation having a planar interface. Further, we assume that the solar wind and the magnetospheric medium carry homogeneous constant magnetic fields which may be unequal, leading to a current sheet at the interface.

We shall regard the solar wind and the magnetospheric medium as continuum fluids. This assumption should be reasonable, as the proton Larmor radius in the interplanetary magnetic field can be shown to be small compared with a typical dimension characterizing the magnetosphere [*Axford*, 1962].

EQUATIONS OF THE PROBLEM

Consider a system of Cartesian axes with the z direction vertical. Suppose that a plane surface of discontinuity of tangential velocity (vortex sheet) and of magnetic field (current sheet) exists at the common interface ($z = 0$) between two semi-infinitely extended homogeneous nondissipative plasmas. Let the uniform velocities and the uniform magnetic fields (both in the horizontal plane) be $\mathbf{U}_1$, $\mathbf{U}_2$ and $\mathbf{H}_1$, $\mathbf{H}_2$, respectively, for $z < 0$ and $z > 0$ regions. Let ρ_1 and ρ_2 denote the uniform densities of the lower and the upper plasmas characterized by sound speeds C_1 and C_2, respectively.

The steady state of the configuration requires

$$\nabla p = 0 \tag{1}$$

for either medium. Here p denotes the plasma pressure. Further, the equilibrium of the interface is governed by

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$$p_1 + \frac{H_1^2}{8\pi} = p_2 + \frac{H_2^2}{8\pi} \quad (2)$$

To investigate the stability of the initial steady state, let us consider the effect of a small velocity field disturbance **u** with components u, v, w in the x, y, z directions.

We write:

$$\begin{aligned} \mathbf{v} &= [\mathbf{U}_{0x},\ \mathbf{U}_{0y},\ 0] + \mathbf{u}(x, y, z, t) \\ \rho &= \rho_0 + \delta\rho(x, y, z, t) \\ p &= p_0 + \delta p(x, y, z, t) \\ \mathbf{H} &= [H_{0x}, H_{0y}, 0] + \mathbf{h}(x, y, z, t) \end{aligned} \quad (3)$$

where $\delta\rho$, δp, and **h** denote perturbations in density, pressure, and magnetic field, respectively, and the suffix 0 refers to equilibrium values. The perturbations are assumed to be of first order of smallness, so that powers higher than the first and their mutual products are neglected.

Assuming the components of the perturbations to vary with x, y, z, t, as (some functions of z) $\exp[ik_x x + ik_y y + nt]$, we can write the linearized hydromagnetic equations as:

$$\rho_0(n + i\mathbf{k}\cdot\mathbf{U}_0)u = -ik_x\delta p - \frac{H_{0y}}{4\pi}(ik_x h_y - ik_y h_x) \quad (4)$$

$$\rho_0(n + i\mathbf{k}\cdot\mathbf{U}_0)v = -ik_y\delta p + \frac{H_{0x}}{4\pi}(ik_x h_y - ik_y h_x) \quad (5)$$

$$\rho_0(n + i\mathbf{k}\cdot\mathbf{U}_0)w = -D\delta p - \frac{H_{0x}}{4\pi}(Dh_x - ik_x h_z) + \frac{H_{0y}}{4\pi}(ik_y h_z - Dh_y) \quad (6)$$

$$(n + i\mathbf{k}\cdot\mathbf{U}_0)\delta\rho = -\rho_0\nabla\cdot\mathbf{u} \quad (7)$$

$$\delta p = C^2\delta\rho \quad (8)$$

$$(n + i\mathbf{k}\cdot\mathbf{U}_0)h_x = i\mathbf{k}\cdot\mathbf{H}_0 u - H_{0x}\nabla\cdot\mathbf{u} \quad (9)$$

$$(n + i\mathbf{k}\cdot\mathbf{U}_0)h_y = i\mathbf{k}\cdot\mathbf{H}_0 v - H_{0y}\nabla\cdot\mathbf{u} \quad (10)$$

$$(n + i\mathbf{k}\cdot\mathbf{U}_0)h_z = i\mathbf{k}\cdot\mathbf{H}_0 w \quad (11)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0 \quad (12)$$

where D stands for d/dz.

Let us now eliminate some of the variables and derive an equation for, say, the vertical component w of velocity perturbation vector. We multiply (4) and (5) by ik_x and ik_y, respectively, and add to obtain

$$\rho_0(n + i\mathbf{k}\cdot\mathbf{U}_0)(\nabla\cdot\mathbf{u} - Dw) + (ik_x h_y - ik_y h_x)\cdot\frac{1}{4\pi}(ik_x H_{0y} - ik_y H_{0x}) = k^2\,\delta p \quad (13)$$

where

$$k^2 = k_x^2 + k_y^2 \quad (14)$$

Eliminating δp from (6) and (13) we get:

$$\begin{aligned} &-\rho_0(n + i\mathbf{k}\cdot\mathbf{U}_0)(D^2 - k^2)w \\ &\quad + \rho_0(n + i\mathbf{k}\cdot\mathbf{U}_0)D\nabla\cdot\mathbf{u} \\ &\quad + \frac{H_{0y}}{4\pi}[ik_x\{ik_x Dh_y - ik_y Dh_x\} \\ &\quad + k^2\{Dh_y - ik_y h_z\}] \\ &\quad - \frac{H_{0x}}{4\pi}[ik_y\{ik_x Dh_y - ik_y Dh_x\} \\ &\quad - k^2\{Dh_x - ik_x h_z\}] = 0 \end{aligned} \quad (15)$$

The magnetic field terms in (15) reduce to, on using (9)–(12),

$$-[(\mathbf{H}_0\cdot\mathbf{k})^2/4\pi(n + i\mathbf{k}\cdot\mathbf{U}_0)](D^2 - k^2)w \quad (16)$$

The perturbation equation 15 is written as

$$\left[1 + \frac{(\mathbf{k}\cdot\mathbf{V}_0)^2}{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2}\right]\cdot(D^2 - k^2)w - D\nabla\cdot\mathbf{u} = 0 \quad (17)$$

where $\mathbf{V}_0$ denotes the Alfvén velocity vector $\mathbf{H}_0/(4\pi\rho_0)^{1/2}$. The expression for $\nabla\cdot\mathbf{u}$ is now obtained from the (7), (8), (9), (10), and (13) as

$$\nabla\cdot\mathbf{u}[C^2k^2 + (n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + (\mathbf{k}\times\mathbf{V}_0)^2] = (n + i\mathbf{k}\cdot\mathbf{U}_0)^2 Dw - \mathbf{k}\cdot\mathbf{V}_0(\mathbf{V}_0\times\mathbf{k})\zeta \quad (18)$$

where ζ is written for $(ik_x v - ik_y u)$.

Let us now evaluate ζ by eliminating δp from (4) and (5). We get

$$\zeta[(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + (\mathbf{k}\cdot\mathbf{V}_0)^2] = -\nabla\cdot\mathbf{u}\,\mathbf{k}\cdot\mathbf{V}_0(\mathbf{k}\times\mathbf{V}_0) \quad (19)$$

Combining (18) and (19), we obtain

$$\nabla\cdot\mathbf{u}[\{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + C^2k^2\}\{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + (\mathbf{k}\cdot\mathbf{V}_0)^2\} + (n + i\mathbf{k}\cdot\mathbf{U}_0)^2(\mathbf{k}\times\mathbf{V}_0)^2]$$
$$= (n + i\mathbf{k}\cdot\mathbf{U}_0)^2\{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + (\mathbf{k}\cdot\mathbf{V}_0)^2\}\,Dw \tag{20}$$

Substituting (20) in (17) we obtain

$$D^2w[\{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + (\mathbf{k}\cdot\mathbf{V}_0)^2\}\cdot\{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + C^2k^2\} + (n + i\mathbf{k}\cdot\mathbf{U}_0)^2(\mathbf{k}\times\mathbf{V}_0)^2 - (n + i\mathbf{k}\cdot\mathbf{U}_0)^4]$$
$$= k^2w[\{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + (\mathbf{k}\cdot\mathbf{V}_0)^2\}\cdot\{(n + i\mathbf{k}\cdot\mathbf{U}_0)^2 + C^2k^2\} + (n + i\mathbf{k}\cdot\mathbf{U}_0)^2(\mathbf{k}\times\mathbf{V}_0)^2] \tag{21}$$

as the equation determining w.

Boundary Conditions and Dispersion Relation

For a configuration of two superposed uniform plasmas slipping past each other at the horizontal interface $z = 0$, the respective solutions, vanishing at $z = \pm\infty$, of (21) are written as:

$$\begin{aligned} w_1 &= A_1\, e^{m_1 z} \qquad (z < 0) \\ w_2 &= A_2\, e^{-m_2 z} \qquad (z > 0) \end{aligned} \tag{22}$$

where

$$m_j^2 = k^2\,\frac{[n_j^2 + (\mathbf{k}\cdot\mathbf{V}_j)^2][n_j^2 + k^2C_j^2] + n_j^2(\mathbf{k}\times\mathbf{V}_j)^2}{[\{n_j^2 + (\mathbf{k}\cdot\mathbf{V}_j)^2\}\{n_j^2 + k^2C_j^2\} + n_j^2(\mathbf{k}\times\mathbf{V}_j)^2 - n_j^4]} \tag{23}$$

Here n_j is written for $n + i\mathbf{k}\cdot\mathbf{U}_j$ for brevity.

At the common interface we have to satisfy the following boundary conditions:

1. The normal component of velocity is continuous; this condition leads to

$$w_1 - U_{1x}\frac{\partial\xi}{\partial x} - U_{1y}\frac{\partial\xi}{\partial y} = w_2 - U_{2x}\frac{\partial\xi}{\partial x} - U_{2y}\frac{\partial\xi}{\partial y} = \frac{\partial\xi}{\partial t} \tag{24}$$

where ξ denotes the small displacement of the interface. Using (22) we get

$$A_2 = A_1 n_2/n_1 \tag{25}$$

2. The normal component of the magnetic field is continuous. We can easily verify that this condition is automatically satisfied as a consequence of condition 1.

3. The normal stress should be continuous across the interface. This means that

$$\delta p_1 - \delta p_2 + \frac{1}{4\pi}[\mathbf{H}_1\cdot(\mathbf{h})_1 - \mathbf{H}_2\cdot(\mathbf{h})_2] = 0 \tag{26}$$

When we use (11), (12), (13), (20), and (25) and simplify, (26) gives, in (27):

$$\rho_2 m_2[n_2^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2]\,\frac{[\{n_2^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2\}\{n_2^2 + k^2C_2^2\} + n_2^2(\mathbf{k}\times\mathbf{V}_2)^2 - n_2^4]}{[\{n_2^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2\}\{n_2^2 + k^2C_2^2\} + n_2^2(\mathbf{k}\times\mathbf{V}_2)^2]}$$
$$+ \rho_1 m_1[n_1^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2]\,\frac{[\{n_1^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2\}\{n_1^2 + k^2C_1^2\} + n_1^2(\mathbf{k}\times\mathbf{V}_1)^2 - n_1^4]}{[\{n_1^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2\}\{n_1^2 + k^2C_1^2\} + n_1^2(\mathbf{k}\times\mathbf{V}_1)^2]} = 0 \tag{27}$$

Substituting the expressions for m_1, m_2 as given by (23), we finally obtain the dispersion relation in the following form:

$$\rho_2[n_2^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2]\,\frac{[\{n_2^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2\}\{n_2^2 + k^2C_1^2\} + n_2^2(\mathbf{k}\times\mathbf{V}_2)^2 - n_2^4]^{1/2}}{[\{n_2^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2\}\{n_2^2 + k^2C_2^2\} + n_2^2(\mathbf{k}\times\mathbf{V}_2)^2]^{1/2}}$$
$$+ \rho_1[n_1^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2]\,\frac{[\{n_1^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2\}\{n_1^2 + k^2C_1^2\} + n_1^2(\mathbf{k}\times\mathbf{V}_1)^2 - n_1^4]^{1/2}}{[\{n_1^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2\}\{n_1^2 + k^2C_1^2\} + n_1^2(\mathbf{k}\times\mathbf{V}_1)^2]^{1/2}} = 0 \tag{28}$$

Equation 28 is the characteristic equation for n, the parameter determining the stability of the configuration. It is rather unwieldy for discussion in the general case, and we shall therefore discuss some special cases, assuming $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{U}_1$, $\mathbf{U}_2$ to be parallel vectors. The assumption should be reasonable, as the solar wind drags the solar-magnetic field with it during radial expansion.

Discussion of Results

Propagation transverse to the direction of streaming ($k_x = 0$, $k_y = k$). If the streaming plasmas carry the same magnetic field (i.e., $\mathbf{H}_1 = \mathbf{H}_2$) and are characterized by the same γ (ratio of the two specific heats), the equilibrium of the interface (equation 2) gives

$$\rho_1 C_1^{\,2} = \rho_2 C_2^{\,2} \tag{29}$$

Also

$$\rho_1 V_1^{\,2} = \rho_2 V_2^{\,2} \tag{30}$$

The dispersion equation, when the wave vector is nonvanishing in the direction transverse to streaming alone, reduces to

$$\rho_2^{\,2}\left[1 - \frac{U_p^{\,2}}{C_1^{\,2} + V_1^{\,2}}\right] = \rho_1^{\,2}\left[1 - \frac{U_p^{\,2}}{C_2^{\,2} + V_2^{\,2}}\right] \tag{31}$$

where U_p is the phase velocity of disturbance, given by in/k.

The configuration of two slipping plasmas is therefore stable, as can be easily verified, for perturbations characterized by $k_x = 0$, $k_y = k$. The perturbation, having a nonvanishing wave number k_x along the direction of streaming may, however, bring about instability. Let us therefore investigate the question of the stability of the configuration for the other extreme case, $k_x = k$, $k_y = 0$.

Propagation parallel to the direction of streaming ($k_x = k$, $k_y = 0$). In this particular case, the general dispersion equation 28 reduces to

$$\rho_2\left[\frac{V_2^{\,2} - (U_2 - U_p)^2}{C_2^{\,2} - (U_2 - U_p)^2}\right]^{1/2}[C_2^{\,2}V_2^{\,2} - (U_2 - U_p)^2(C_2^{\,2} + V_2^{\,2})]^{1/2}$$
$$= -\rho_1\left[\frac{V_1^{\,2} - (U_1 - U_p)^2}{C_1^{\,2} - (U_1 - U_p)^2}\right]^{1/2}[C_1^{\,2}V_1^{\,2} - (U_1 - U_p)^2(C_1^{\,2} + V_1^{\,2})]^{1/2} \tag{32}$$

where C_1, C_2, and V_1, V_2 are related by (29) and (30), respectively. If both C_1 and C_2 tend to infinity, the above equation gives the dispersion formula for Kelvin-Helmholtz instability for incompressible fluids in hydromagnetics. This is the same equation obtained and discussed in earlier papers [*Talwar*, 1961, 1962].

To simplify the discussion still further, let us take $U = -U_2 = +U_1$. Equation 32 is then written as:

$$\begin{aligned}
&U_p^{\,6}(1 - \delta) - 2UU_p^{\,5}(1 + \delta) \\
&\quad - U_p^{\,4}(1 - \delta)[U^2 + (1 + \delta)C_2^{\,2}] + 4UU_p^{\,3}[U^2(1 + \delta) + C_2^{\,2}(1 + \delta^2)] \\
&\quad + U_p^{\,2}(1 - \delta)\left[-U^4 - 6U^2C_2^{\,2}(1 + \delta) + 2\delta\frac{C_2^{\,4}V_2^{\,2}}{(C_2^{\,2} + V_2^{\,2})}\right] \\
&\quad + U_p\left[-2U^5(1 + \delta) + 4U^3C_2^{\,2}(1 + \delta^2) - \frac{4UC_2^{\,4}V_2^{\,2}}{(C_2^{\,2} + V_2^{\,2})}\delta(1 + \delta)\right] \\
&\quad + (1 - \delta)\left[U^6 - C_2^{\,2}U^4(1 + \delta) + \frac{2U^2\delta C_2^{\,4}V_2^{\,2}}{(C_2^{\,2} + V_2^{\,2})}\right] = 0
\end{aligned} \tag{33}$$

Here $\delta = \rho_2/\rho_1$, and we have used (29) and (30). Equation 33 is a sixth degree equation in phase velocity, and to insure that the configuration of two slipping plasmas is stable also for perturbations along the direction of streaming, we require that all the roots of the above equation should be real. In a particularly simple case, where we take $\delta = 1$ so that different layers of the same plasma fluid permeated with a uniform horizontal magnetic field have a relative tangential velocity, (33) gives:

$$U_p^{\,4} - 2U_p^{\,2}(C^2 + U^2) + \left[U^4 - 2U^2C^2 + \frac{2C^4V^2}{C^2 + V^2}\right] = 0 \tag{34}$$

where now $C_1 = C_2 = C$ and $V_1 = V_2 = V$.

Equation 33 gives the following condition for the stability of the configuration

$$U > C\left[1 + \left(\frac{C^2 - V^2}{C^2 + V^2}\right)^{1/2}\right]^{1/2} \tag{35}$$

Here $2U$ denotes the relative speed between two layers of the plasma characterized by the sound speed C and hydromagnetic speed V. In the absence of a magnetic field, we get the condition for stability $U > \sqrt{2}C$. We thus conclude that, whereas a tangential discontinuity in a liquid is unstable for all nonzero speeds in the absence of magnetic field, it is no longer so unless the relative speed is smaller than a certain critical value in a compressible fluid. In the presence of magnetic fields the stability of the vortex sheet in compressible plasmas is improved (as also is the case with infinitely conducting liquids) in that the critical relative speed below which the configuration is unstable is decreased as a consequence of the magnetic field. Alternatively, we could conclude that any relative speed between two layers of plasma can be stabilized by a strong enough magnetic field given by:

$$V_*^2 = \frac{U^2C^2(2C^2 - U^2)}{(U^2 - C^2)^2 + C^4} \qquad (36)$$

Figures 1 and 2, respectively, show the dependence of critical tangential velocity U^* in units of C as a function of V/C and that of critical Alfvén speed against U^2/C^2.

The above results should have a useful application in determining whether the interface of the magnetosphere with the solar wind is stable or not. Observations [*Gringauz et al.*, 1960; *Bridge et al.*, 1961] show that the solar wind has a speed ~250–400 km/sec and density ~10

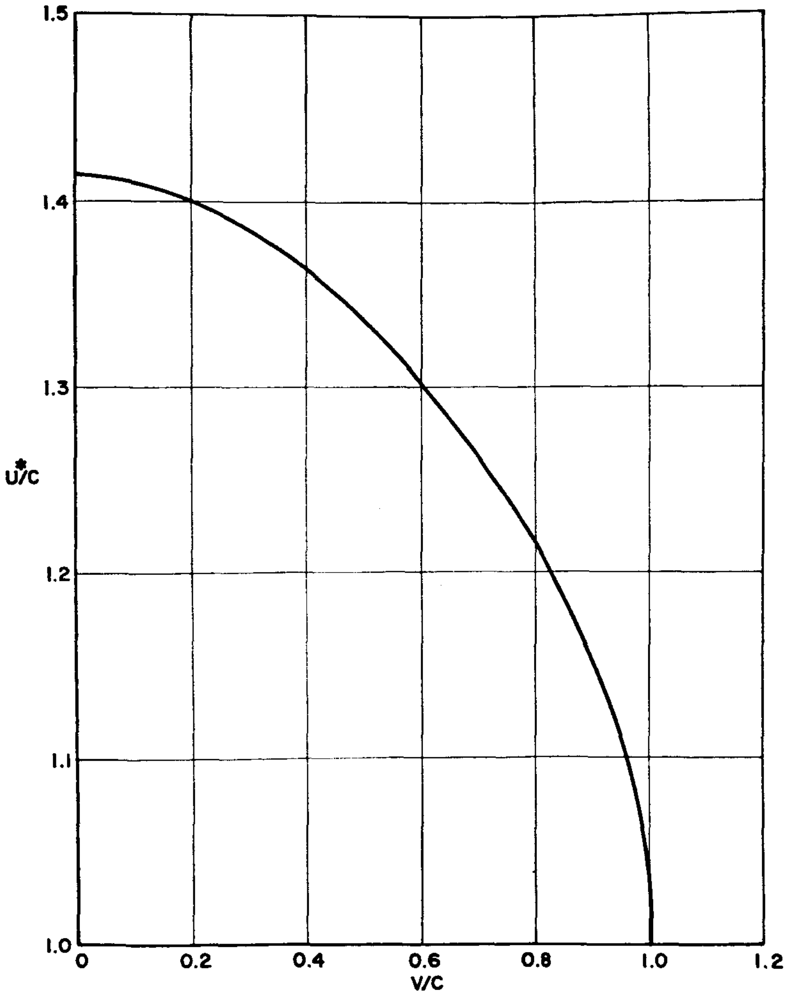

Fig. 1. A plot of the critical tangential velocity U^* as measured in units of C, against V/C, the ratio of the Alfvén speed to the sound speed.

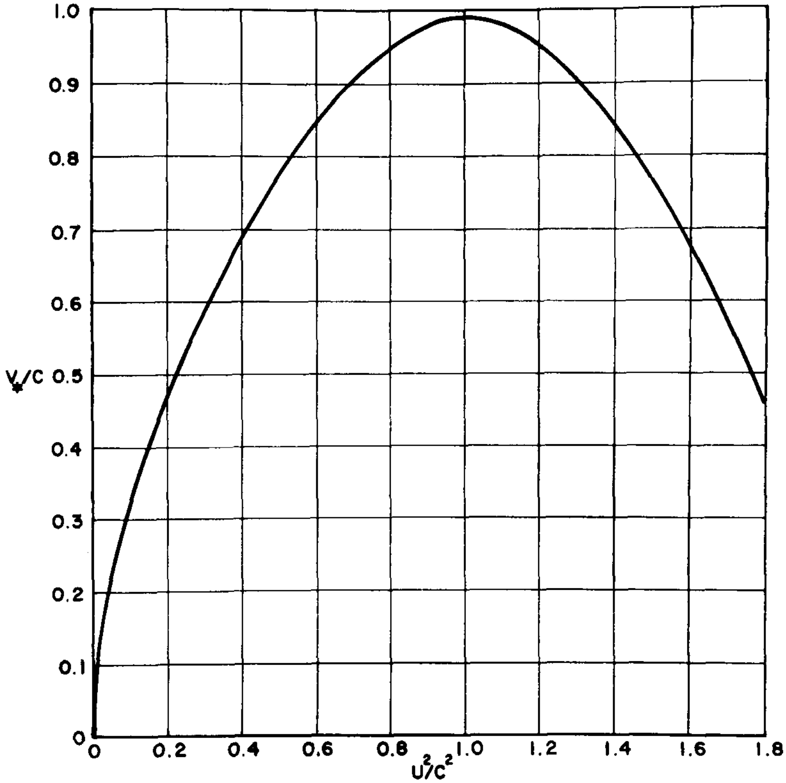

Fig. 2. The dependence of the critical Alfvén speed V_* against U^2/C^2, the square of the ratio of the tangential speed to the sound speed.

protons per cc under quiet solar conditions. When the sun is active, these values may mount to $\sim 10^3$ km/sec with densities ~ 300 per cc. The speed of the incoming solar wind is therefore supersonic as it hits the magnetosphere. As the solar wind streams along the magnetospheric boundary, the speed of solar plasma relative to the medium within the magnetospheric boundary is likely to diminish in value. This may be on account of internal motions within the magnetosphere [*Gold,* 1959]. The internal convective motions (roughly along the magnetospheric boundary) may arise as a result of viscous-like interaction between the magnetosphere and the solar wind [*Axford and Hines,* 1961]. If this relative speed is continuously diminished as the solar wind sweeps around nrom the day side to the night side of the magfetosphere, it may very well happen that the relative speed at some place on the interface becomes less than the critical value given by (34), and that part of the interface thereafter becomes unstable. This situation leading to the instability of the magnetosphere–solar wind interface is likely to arise toward the tail of the magnetosphere and under quiet solar conditions when the speed of the incoming solar wind is relatively small. A verification of the above ideas is possible if it can be ascertained that the characteristic effects of instability, e.g. turbulence, high-level geomagnetic fluctuations, and irregularities of ionization density, are observed during comparatively quiet solar conditions and more so toward the night side.

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REFERENCES

Axford, W. I., The interaction between the solar wind and the earth's magnetosphere, *J. Geophys. Res., 67,* 3791–3796, 1962.

Axford, W. I., and C. O. Hines, A unifying theory of high-latitude geophysical phenomena and geomagnetic storms, *Can. J. Phys., 39,* 1433–1464, 1961.

Bridge, H. S., C. Dilworth, A. J. Lazarus, E. F. Lyon, and B. Rossi, Direct observations of the interplanetary plasma, *Intern. Conf. Cosmic Rays Earth Storm,* Physical Society of Japan, Kyoto, Japan, 1961.

Dessler, A. J., The stability of the interface between the solar wind and the geomagnetic field, *J. Geophys. Res., 66,* 3587–3590, 1961.

Dungey, J. W., Electrodynamics of the outer atmosphere, *Proc. Ionosphere Conf.,* pp. 229–236, The Physical Society of London, 1955.

Gold, T., Motions in the magnetosphere of the earth, *J. Geophys. Res., 64,* 1219–1224, 1959.

Gringauz, K. I., V. V. Bezrukikh, V. D. Ozerov, and R. E. Rybchinskii, A study of interplanetary ionized gas, energetic electrons, and solar corpuscular radiation using three-electrode charged particle traps on the second Soviet cosmic rocket, *Dokl. Akad. Nauk SSSR, 131,* 1301, 1960.

Parker, E. N., Interaction of the solar wind with the geomagnetic field, *Phys. Fluids, 1,* 171–187, 1958.

Parker, E. N., *Interplanetary Dynamical Processes,* Interscience Publishers, New York, 1963.

Talwar, S. P., Helmholtz instability in hydromagnetics, *Proc. Natl. Inst. Sci., India, 27,* 263–268, 1961.

Talwar, S. P., Helmholtz instability in hydromagnetics, 2, *Indian J. Phys., 36,* 351–358, 1962.

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